§20. Collisionless Damping of Zonal Flows in Helical Systems

Sugama, H., Watanabe, T.-H.

We here consider helical systems with the magnetic field strength written by a function of poloidal and toroidal angles as $B = B_0[1 - \epsilon_{10}\cos\theta - \epsilon_{L0}\cos(L\theta) - \sum_{|n|\le n_{max}} \epsilon_h^{(n)} \cos\{(L+n)\theta - M\zeta\}] = B_0[1 - \epsilon_T(\theta) - \epsilon_H(\theta)\cos\{L\theta - M\zeta + \chi_H(\theta)\}]$. Using the gyrokinetic equations combined with the quasineutrality condition, an analytical expression describing the collisionless response of the zonal-flow potential to the initial potential is derived [1,2] as

$$\phi_{\mathbf{k}_\perp}(t) = \mathcal{K}(t)\,\phi_{\mathbf{k}_\perp}(0) \qquad (1)$$

where the response function (or kernel) $\mathcal{K}(t)$ is written as

$$\mathcal{K}(t) = \mathcal{K}_{GAM}(t)[1 - \mathcal{K}_L(t)] + \mathcal{K}_L(t). \qquad (2)$$

The long-time response is denoted by $\mathcal{K}_L(t)$ that gives the residual zonal-flow level in the limit $t \to \infty$. The short-time response $\mathcal{K}_{GAM}(t)$ represents the GAM oscillations given by

$$\mathcal{K}_{GAM}(t) = \cos(\omega_G t)\exp(\gamma t). \qquad (3)$$

with $\omega = \omega_G + i\gamma$ detemined by $1/\mathcal{K}_{GAM}(\omega) = 0$. Here, $\mathcal{K}_{GAM}(\omega)$ is defined by

$$\begin{aligned}\frac{1}{\mathcal{K}_{GAM}(\omega)} \equiv\ & -i\hat{\omega} - i\frac{q^2}{2}\left[\left(\frac{R_0\epsilon_{10}}{r}\right)^2 \{J(\hat{\omega}) + J_{\mathrm{FOW}}(\hat{\omega})\}\right.\\ & + L\left(\frac{R_0\epsilon_{L0}}{r}\right)^2 J\left(\frac{\hat{\omega}}{L}\right)\\ & + \sum_{|n|\le n_{max}} \frac{(L+n)^2}{|L+n-qM|}\left(\frac{R_0\epsilon_h^{(n)}}{r}\right)^2\\ & \left.\times J\left(\frac{\hat{\omega}}{|L+n-qM|}\right)\right], \qquad (4)\end{aligned}$$

with $\hat{\omega} \equiv R_0 q\omega/v_{Ti}$ ($v_{Ti} \equiv \sqrt{2T_i/m_i}$),

$$\begin{aligned}J(\hat{\omega}) \equiv\ & 2\hat{\omega}^3 + 3\hat{\omega} + (2\hat{\omega}^4 + 2\hat{\omega}^2 + 1)Z(\hat{\omega})\\ & - \frac{\hat{\omega}}{2}\left\{2\hat{\omega} + (2\hat{\omega}^2+1)Z(\hat{\omega})\right\}^2\\ & \times\left\{\frac{T_i}{T_e} + 1 + \hat{\omega}Z(\hat{\omega})\right\}^{-1}, \qquad (5)\end{aligned}$$

and

$$\begin{aligned}J_{\mathrm{FOW}}(\hat{\omega}) \equiv\ & i\frac{\sqrt{\pi}}{2}\left(\frac{k_r v_{Ti} q}{\Omega_i}\right)^2 e^{-\hat{\omega}_r^2/4}\\ & \times\left\{\frac{\hat{\omega}_r^6}{64} + \left(\frac{\hat{\omega}_r^4}{8} + \frac{3\hat{\omega}_r^2}{4} + 3 + \frac{6}{\hat{\omega}_r^2}\right)\right.\\ & \times\left(1 - \frac{3\hat{\omega}_r}{16}\left\{2\hat{\omega}_r + (2\hat{\omega}_r^2+1)Z_r(\hat{\omega}_r)\right\}\right.\\ & \left.\left.\times\left\{\frac{T_i}{T_e} + 1 + \hat{\omega}_r Z_r(\hat{\omega}_r)\right\}^{-1}\right)\right\}. \qquad (6)\end{aligned}$$

On the right-hand side of Eq. (5), the plasma dispersion function $Z(\hat{\omega}) \equiv \pi^{-1/2}\int_{-\infty}^{\infty} d\alpha\, e^{-\alpha^2}/(\alpha - \hat{\omega})$ is used. J_{FOW} given in Eq. (6) is derived from retaining the finite-orbit-width (FOW) effect on the gyrocenter distribution function. The validity of our analytical results is verified by gyrokinetic Vlasov simulation as shown in Fig.1. It is found that helical ripples in the magnetic field strength as well as finite orbit widths of passing ions enhance the GAM damping.

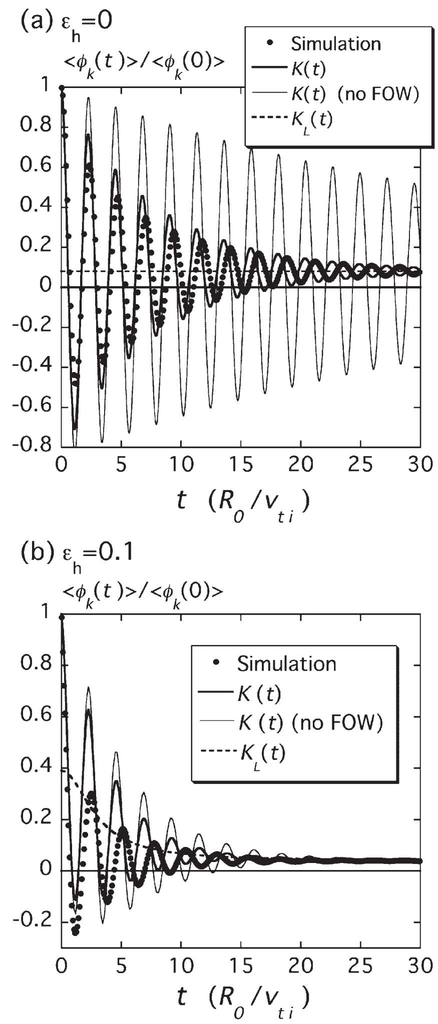

Fig.1. Time evolution of the zonal-flow potential obtained by the simulations for the tokamak case ($\epsilon_h = 0$) (a) and for for the helical system ($\epsilon_h = 0.1$) (b). In both case, $r/R_0 = 0.1$, $q = 1.5$ and $k_r a_i = 0.131$ are used. The simulation results are plotted by solid circular symbols. Thick solid curves represent the response kernel $\mathcal{K}(t)$ obtained by Eq. (2) with the use of the complex-valued GAM frequency $\omega = \omega_G + i\gamma$ calculated by numerically solving $1/\mathcal{K}_{GAM}(\omega) = 0$ where $\mathcal{K}_{GAM}(\omega)$ is defined by Eq. (4). Thin solid curves represent the response kernel $\mathcal{K}(t)$ obtained by neglecting the FOW effect when calculating (ω_G, γ). The long-time response kernel $\mathcal{K}_L(t)$ is also plotted by dashed lines.

References

1) H. Sugama and T.-H. Watanabe, Phys. Plasmas **13**, 012501 (2006).
2) H. Sugama and T.-H. Watanabe, Phys. Rev. Lett. **94**, 115001 (2005).